Complex mixed dark-bright wave patterns to the modified α and modified Vakhnenko-Parkes equations

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Abstract In this paper, we present the sine-Gordon expansion method to prepare the mixed dark bright wave patterns to the nonlinear partial differential equations arising in mathematical physics. Then, we apply the proposed method for a credible recourse of two nonlinear physical models: the modified Vakhnenko-Parkes and modified α-equation. These exact solutions comprise the hyperbolic, trigonometric, rational and exponential function with few licentious parameter. The analytical solutions have different physical structures and they are graphically analyzed in order to show their dynamical behavior by means of 2D, 3D and contour plots.

Keywords: Modified Vakhnenko-Parkes Equation; Modified α-equation; sine-Gordon expansion method; Complex hyperbolic, trigonometric, rational and exponential function solutions
2010 Mathematics Subject Classification: 35Axx; 35Lxx; 35Qxx

1 Introduction

Nonlinear partial differential equations (NPDEs) are used to explain nonlinear complex physical occurrence, which play a vital role in physics and appear in several fields of science and engineering. Travelling wave solutions of NPDEs play a vital role in the visibility of these physical phenomena in nature such as vibrations, self-strong and so on. Finding new solutions for NPDEs is a major and important work that plays an important role in topology. Over the last few years, exact solution, analytical approximate solution, and numerical solution of many NPDEs have been proudly extracted attentions of expert from all over the world. Some powerful methods which have been newly evolved to quest exact explicit solution of NPDEs are, for example, the simplified hirota's technique [1, 4], the variational iteration method [2], the finite forward difference method [3], the lie symmetry method [5, 10, 29], the generalized riccati mapping method [6], the tanh method and SCM [7, 40], the fourth-order iterative method [8], the sine-Gordon expansion method [9, 14, 16, 24, 42, 43], the hirota's bilinear form method [11, 23], compact envelope dark solitary wave[12], the tanh and extended tanh method [13, 38, 40], the JEFM [15]-[36], the tanh function method [17, 34, 38], the modified $\exp(-\Omega(\xi))$-expansion method [18]-[22], dark-brite soliton solution[19], the MSEM and the ESEM [20]-[30], the matrix analysis method[21], the via improved bernoulli sub equation method[25], the $(\frac{G'}{G})$expansion approach [26], the bifurcation method [28], the generalized darboux transfor [31]-[33], the finite difference method [32],the MESM [35], and many others. The rest of this paper is consolidated follows, the regular property of the described method is given in Section 2. To obtain the many new soliton solutions of MAE and MVPE, SGEM is applied. The 2D, 3D and contour simulations of the new solution are plotted in Section 4. Finally, we give a conclusion in a detailed manner.

2 The SGEM

In this section, we present the (SGEM) as following [9, 14, 16, 24, 42];

$$u_{xx} - u_{tt} = m^2 sin(u), \tag{1}$$

where $u = u(x,t)$, m is a real const. Applying the wave transform $u(x,t) = U(\xi)$, $\xi = x - ct$ to Eq.(3), we get the following NLODEs;

$$U'' = \frac{m^2}{(1-c^2)} sin(U), \tag{2}$$

where $U = U(\xi)$, ξ and c are the dimension and velocity of the traveling waves, respectively. We integrate Eq.(4) and it can be inscribed as follows;

$$\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{m^2}{(1-c^2)} sin^2\left(\frac{U}{2}\right) + K, \tag{3}$$

where K is the constant of integration. Substituting $K = 0, \omega(\xi) = \frac{U}{2}$ and $a^2 = \frac{m^2}{(1-c^2)}$ in Eq.(5), it yields

$$\omega' = asin(\omega). \tag{4}$$

Setting $a = 1$ in Eq.(6) gives

$$\omega' = sin(\omega). \tag{5}$$

Solving Eq.(7) by variable separable, we receive the two important properties as

$$sin(\omega) = sin(\omega(\xi)) = \left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1} = sech(\xi), \tag{6}$$

$$cos(\omega) = cos(\omega(\xi)) = \left.\frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\right|_{p=1} = tanh(\xi), \tag{7}$$

where p is an integral const. and non-zero. By using these vital, properties, we can consider the general form of NPDEs as

$$P(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt,\ldots}) = 0, \tag{8}$$

where $u = u(x,t)$. We consider the solutions of Eq.(10) as following expression,

$$U(\xi) = \sum_{i=1}^{n} tanh^{i-1}(\xi)[B_i sech(\xi) + A_1 tanh(\xi)] + A_0. \tag{9}$$

Eq.(11) can be rearranged ethically to Eq.(8) and Eq.(9) as follows;

$$U(\omega) = \sum_{i=1}^{n} cos^{i-1}(\omega)[B_i sin(\omega) + A_1 cos(\omega)] + A_0. \tag{10}$$

Using the homogenous equilibrium theory to determine the value of n is considered. We suppose that the sum of coefficient of $sin^i(\omega)cos^j(\omega)$ with the equal strength is naught, this yields an equation arrangement. With aid of the computational program, we solve the equation system to find the value of A_i, B_i, μ and c. Finally, substitute the values of A_i, B_i, μ and c into Eq.(11), we get the recent traveling wave solution to the Eq.(10).

3 Applications and Mathematical Analysis

In this part of the paper, we investigate two models afternamed modified α equation (MAE) defined as

$$u_t - u_{xxt} + (\alpha+1)u^2u_x - \alpha u_x u_{xx} - uu_{xxx} = 0, \tag{11}$$

In 2006 and 2019 Islam et al. and Wazwaz studied on a family physical properties of Eq.(11) in [35], in which α is a positive integer, Eq.(11) is a strategic application for delineate the procedure of phase dissociation: in cold steel alloy and ordinarily used in solidifying and nucleation problem where in $u(x,t)$ the two independent variables are an unnamed function of x and t that indicate the space variables in the flank of wave publicity and time, respectively. The unnamed function $u(x,t)$ denotes the dimension of the relevant wave mode, the terms u^2u_x and uu_{xxx} denote the nonlinear wave steepening and u_{xxt} denotes the disbandment wave effects. The coefficients α is a positive integer Eq.(11) many well-know nonlinear wave equation can be diminished.

Secondly, modified Vakhnenko-Parkes model (MVPE) defined as [1]

$$uu_{xxt} - u_x u_{xt} + u^3 u_t = 0, \tag{12}$$

is considered. MVPE has been newly and firstly introduced by Wazwaz in 2019 [1]. He has proved that the MVPE satisfy the Painleve properties. More recently, S.Sakovich has shown that MVPE bears the features of sine-Gordon equation [7]. In this paper, we study to find new hyperbolic function solution of MAE and MVPE by using SGEM based on sine-Gordon equation firstly. It is reasonable that many models in science and engineering have an empirically parameter. Thus, unspoiled solution give freedom to researchist to structure and dash experiment, by establish suitable or inartificial condition, to regulate these parameter. Therefore, explication and receive exact travelling wave solution is becoming copious seductive in nonlinear sciences.

3.1 Investigations of MAE

Using the traveling contemplate the wave transformations

$$u(x,t) = U(\xi), \quad \xi = kx - ct, \tag{13}$$

where k and c are real const. and non-zero. Substituting Eq.(13) into Eq.(11), the following nonlinear differential equations is obtained :

$$ck^2U''' - k^3UU''' + (\frac{k^3 - \alpha k^3}{2})(U')^2 - cU' + \frac{k}{3}(\alpha+1)U^3 = 0, \tag{14}$$

Integrate Eq.(14) first with regard to ξ and situation the constant of integrate to zero, yields following NODE

$$6ck^2U'' - 6k^3UU'' + 3k^3(1-\alpha)(U')^2 - 6cU + 2k(\alpha+1)U^3 = 0, \tag{15}$$

Balancing in Eq.(15), it yields as $n = 2$. Then, we get the follows

$$U(w) = B_1 sin(w) + A_1 cos(w) + B_2 cos(w) sin(w) + A_2 cos^2(w) + A_0, \tag{16}$$

differentiating Eq.(16) twice, yields

$$\begin{aligned} U''(w) = & B_1 cos^2(w) sin(w) - B_1 sin^3(w) - 2A_1 sin^2(w) cos(w) + \\ & B_2 cos^3 sin(w) - 5B_2 sin^3(w) cos(w) - 4A_2 cos^2(w) sin^2(w) + \\ & 2A_2 sin^4(w), \end{aligned} \tag{17}$$

Substitute Eqs.(16,17) into Eq.(15) we find a system of equation in the form of trigonometric function through building few trigonometric identities replacement, we can collect a set of algebraically equation by equate every sum of the multiples of the trigonometric function $sin^i(w)cos^i(w)$ with the equal strength to zero to receive the soliton solution of Eq.(11), we replacement the acquired value of the replacement into Eq.(9) by thought $n = 2$.

Case-1 When we consider as $\alpha = 4,\ A_0 = \frac{96}{25} - \frac{3i}{25},\ A_1 = 0,\ A_2 = -\frac{126}{25} + \frac{18i}{25},\ B_1 = 0,\ B_2 = \frac{18}{25} + \frac{126i}{25}.\ k = \sqrt{-\frac{7}{5} + \frac{i}{5}},\ c = \frac{3}{5}\sqrt{\frac{73}{5} + \frac{161i}{5}}$ and inserting these values along with Eq.(13) into Eq.(9), yields following new complex and mixed dark-bright soliton solutions to the MAE as

$$\begin{aligned} u_1(x,t) = & (\frac{96}{25} - \frac{3i}{25}) - (\frac{18}{25} + \frac{126i}{25}) Sech[\frac{3}{5}\sqrt{\frac{73}{5} + \frac{161i}{5}}t - \sqrt{-\frac{7}{5} + \frac{i}{5}}x] \\ & Tanh[\frac{3}{5}\sqrt{\frac{73}{5} + \frac{161i}{5}}t - \sqrt{-\frac{7}{5} + \frac{i}{5}}x] \\ & - (\frac{126}{25} - \frac{18i}{25}) Tanh[\frac{3}{5}\sqrt{\frac{73}{5} + \frac{161i}{5}}t - \sqrt{-\frac{7}{5} + \frac{i}{5}}x]^2. \end{aligned} \tag{18}$$

Case-2 If $\alpha = 4,\ A_0 = \frac{96}{25} + \frac{3i}{25},\ A_1 = 0,\ A_2 = -\frac{126}{25} - \frac{18i}{25},\ B_1 = 0,\ B_2 = \frac{18}{25} - \frac{126i}{25},\ k = \sqrt{-\frac{7}{5} - \frac{i}{5}},\ c = \frac{3}{5}\sqrt{\frac{73}{5} - \frac{161i}{5}}$, putting these together Eq.(13)

into Eq.(9), presents another novel complex soliton to the MAE as

$$u_2(x,t) = (\frac{96}{25} + \frac{3i}{25}) - (\frac{18}{25} - \frac{126i}{25})Sech[\frac{3}{5}\sqrt{\frac{73}{5} - \frac{161i}{5}}t - \sqrt{-\frac{7}{5} - \frac{i}{5}}x]$$
$$Tanh[\frac{3}{5}\sqrt{\frac{73}{5} - \frac{161i}{5}}t - \sqrt{-\frac{7}{5} - \frac{i}{5}}x]$$
$$- (\frac{126}{25} + \frac{18i}{25})Tanh[\frac{3}{5}\sqrt{\frac{73}{5} - \frac{161i}{5}}t - \sqrt{-\frac{7}{5} - \frac{i}{5}}x]^2. \quad (19)$$

Case-3 When $\alpha = 4,\ A_0 = -\frac{18}{5},\ A_1 = 0,\ A_2 = -\frac{18}{5},\ B_1 = 0,\ B_2 = \frac{18i}{5},$ $k = 1,\ c = 3$, it produces following another new mixed dark-bright soliton

$$u_3(x,t) = -\frac{18}{5} - \frac{18}{5}iSech[3t - x]Tanh[3t - x] + \frac{18}{5}Tanh[3t - x]^2. \quad (20)$$

Case-4 If we take as $\alpha = 4,\ A_0 = -\frac{18}{5},\ A_1 = 0,\ A_2 = -\frac{18}{5},\ B_1 = 0,\ B_2 = -\frac{18i}{5},\ k = 1,\ c = 3$, we get another conjugate mixed dark-bright soliton

$$u_4(x,t) = -\frac{18}{5} + \frac{18}{5}iSech[3t - x]Tanh[3t - x] + \frac{18}{5}Tanh[3t - x]^2. \quad (21)$$

Case-5 Considering these values of $\alpha = 4,\ A_0 = -\frac{9}{5},\ A_1 = 0,\ A_2 = \frac{9}{5},\ B_1 = 0,\ B_2 = 0,\ k = \frac{1}{2},\ c = \frac{3}{2}$, it presents new dark soliton solution

$$u_5(x,t) = -\frac{9}{5} + \frac{9}{5}Tanh[\frac{3t}{2} - \frac{x}{2}]^2. \quad (22)$$

Case-6 Taking $\alpha = 6,\ A_0 = -\frac{24}{7},\ A_1 = 0,\ A_2 = \frac{24}{7},\ B_1 = 0,\ B_2 = -\frac{24i}{7},\ k = -1,\ c = -4$, we obtain other solution as

$$u_6(x,t) = -\frac{24}{7} - \frac{24}{7}iSech[4t - x]Tanh[4t - x] + \frac{24}{7}Tanh[4t - x]^2. \quad (23)$$

Case-7 Getting $\alpha = 6,\ A_0 = -\frac{24}{7},\ A_1 = 0,\ A_2 = \frac{24}{7},\ B_1 = 0,\ B_2 = \frac{24i}{7},\ k = -1,\ c = -4$, we gain other conjugate results according to different values of α as

$$u_7(x,t) = -\frac{24}{7} + \frac{24}{7}iSech[4t - x]Tanh[4t - x] + \frac{24}{7}Tanh[4t - x]^2. \quad (24)$$

3.2 Investigation of MVPE

It can be considered the traveling wave transformation for the MVPE Eq.(12) as

$$u(x,t) = U(\xi), \xi = kx - ct. \tag{25}$$

Using above transformation into Eq.(12), we get the following equation;

$$-4k^2 UU'' + 4k^2(U')^2 - U^4 = 0. \tag{26}$$

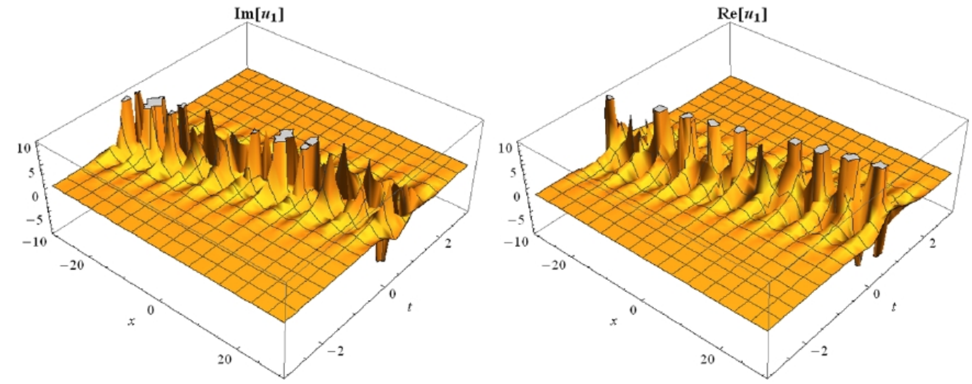
With the balance rule, we find $n = 1$. Using as $n = 1$ into the Eq.(10) gives the following form

$$U(\omega) = B_1 sin(\omega) + A_1 cos(\omega) + A_0. \tag{27}$$

Getting necessary derivations of Eq.(27), if we consider them into Eq.(26), we can find an algebraic system being various coefficients of trigonometric functions. When we solve these systems via various computational programs, we can find $A_o = 0, A_1 = 0, k = -\frac{1}{2}B_1$ which gives the following hyperbolic function solutions as;

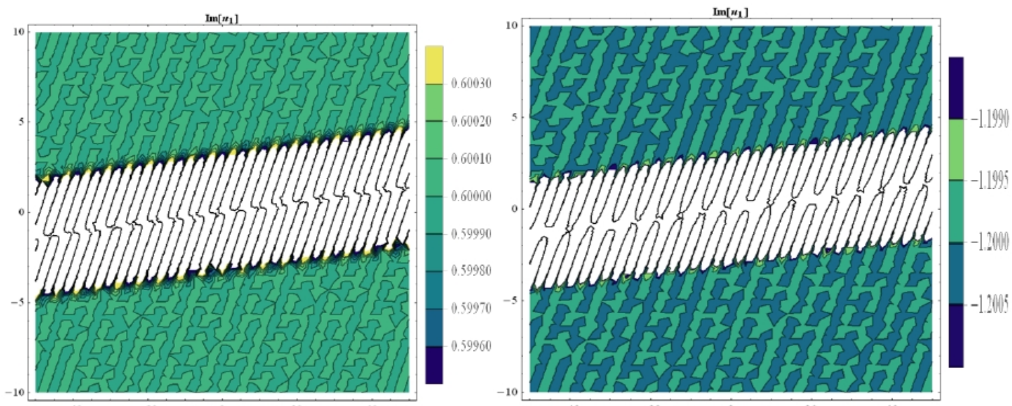
$$u(x,t) = B_1 sech\left(ct + \frac{B_1}{2}x\right). \tag{28}$$

where c and B_1 are real constants with non-zero for valid of solution. Under the suitable values of parameters, we plot several surfaces of solution obtained in this paper by using SGEM.



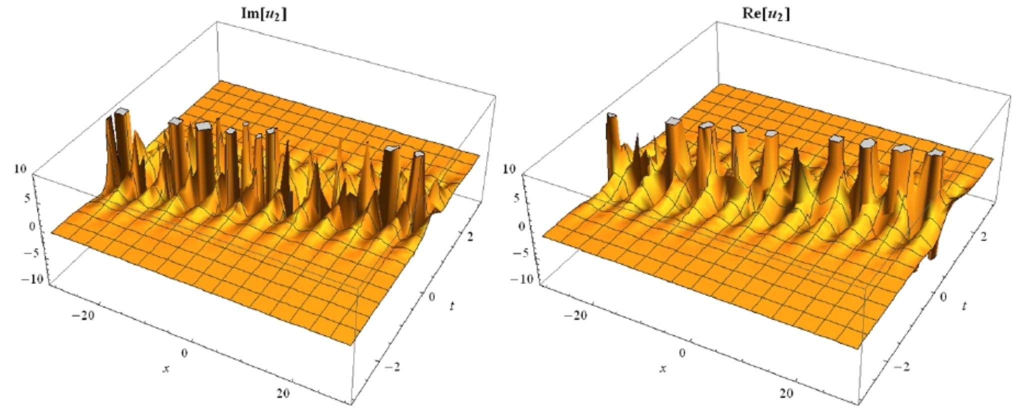
(a) (b)

Fig.1: (a) and (b) represent 3D plots of Eq. (18) for α = 4.



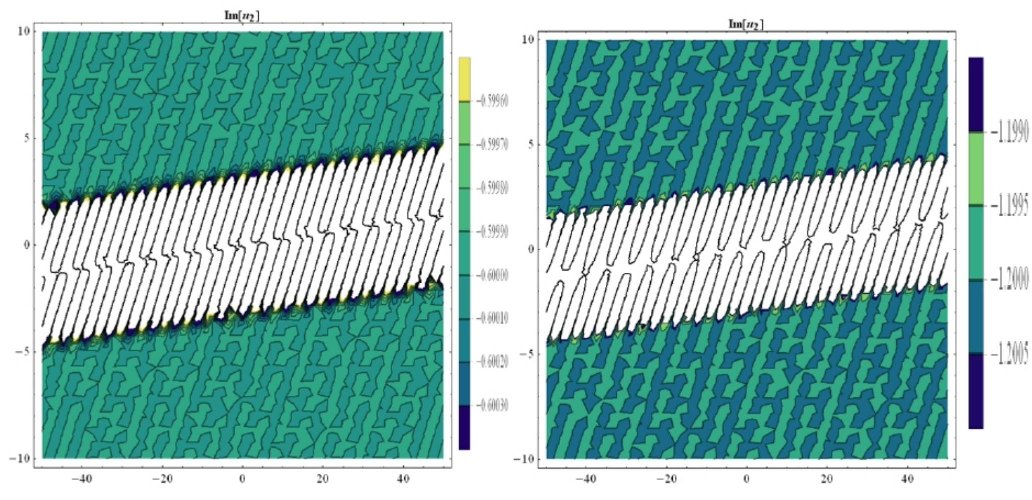
(c) (d)

Fig.2: (c) and (d) represent contour plots of Eq. (18) for α = 4.



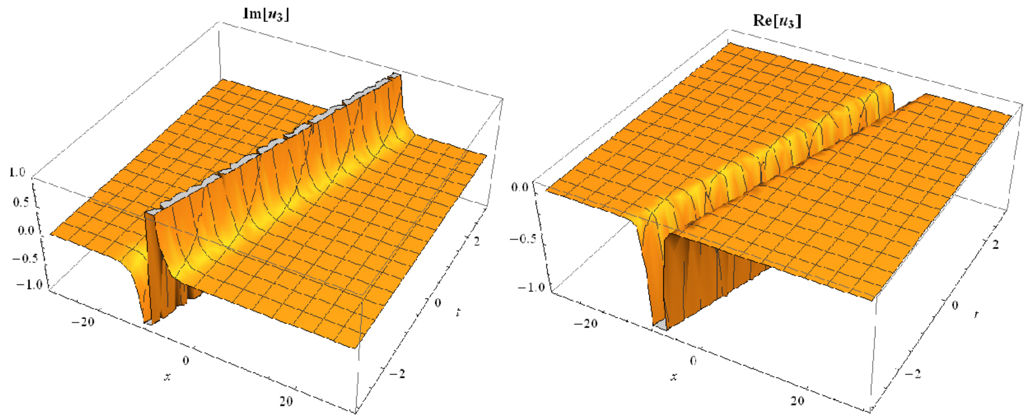
(a) (b)

Fig.3: (a) and (b) represent 3D plots of Eq. (19) for α = 4.



(c) (d)

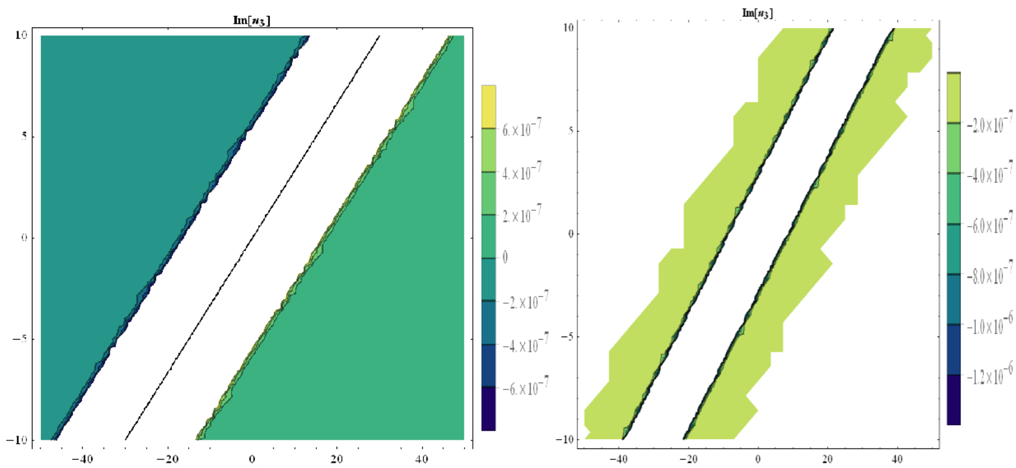
Fig.4: (c) and (d) represent contour plots of Eq. (19) for α = 4.



(a) (b)

Fig.5: (a) and (b) represent 3D plots of Eq. (20) for $\alpha = 4$

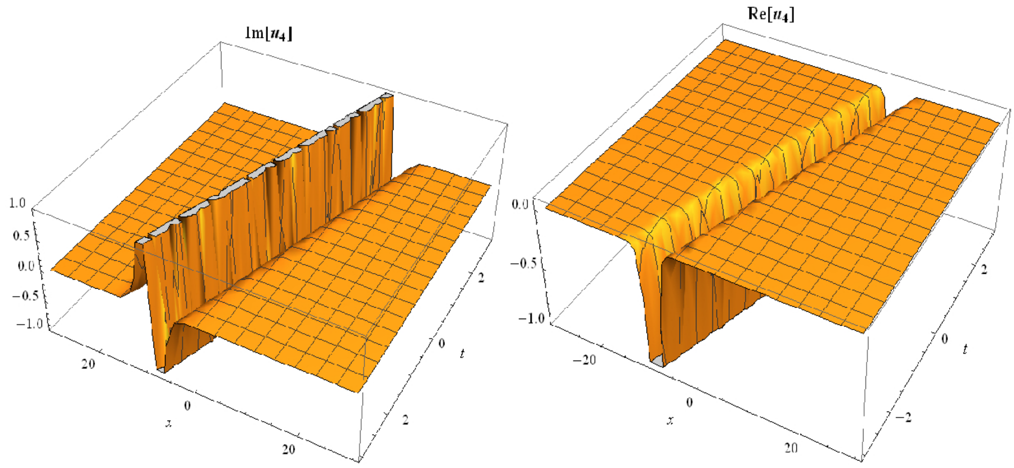
$k = 1,\ c = 3.$



(c) (d)

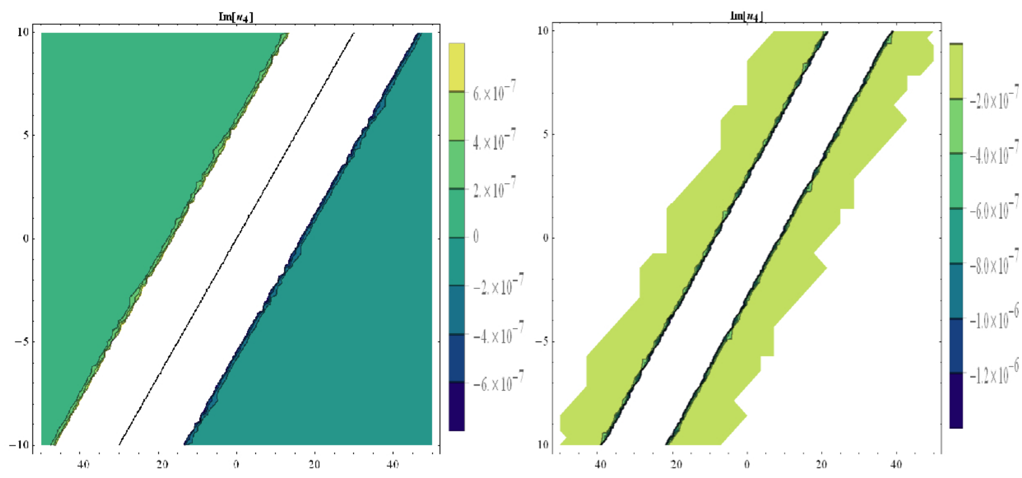
Fig.6: (c) and (d) represent contour plots of Eq. (20) for $\alpha = 4.$

$k = 1,\ c = 3.$



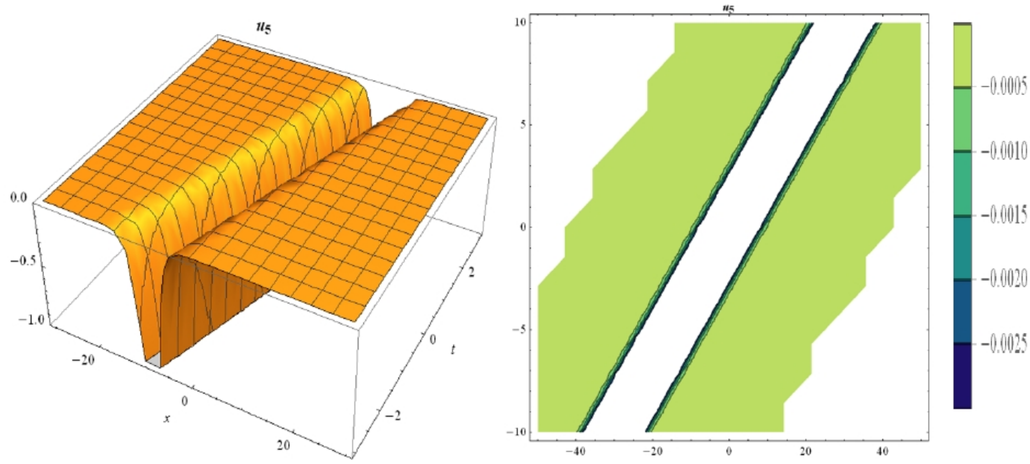
(a) (b)

Fig.7: (a) and (b) represent 3D plots of Eq. (21) for $\alpha = 4$, $k = 1$, $c = 3$.



(c) (d)

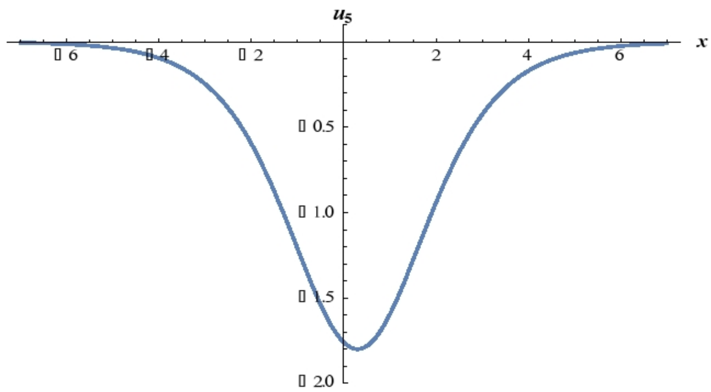
Fig.8: (c) and (d) represent contour plots of Eq. (21) for $\alpha = 4$, $k = 1$, $c = 3$.



(a) (b)

Fig.9: (a) and (b) represent 3D and contour plots of Eq. (22) for

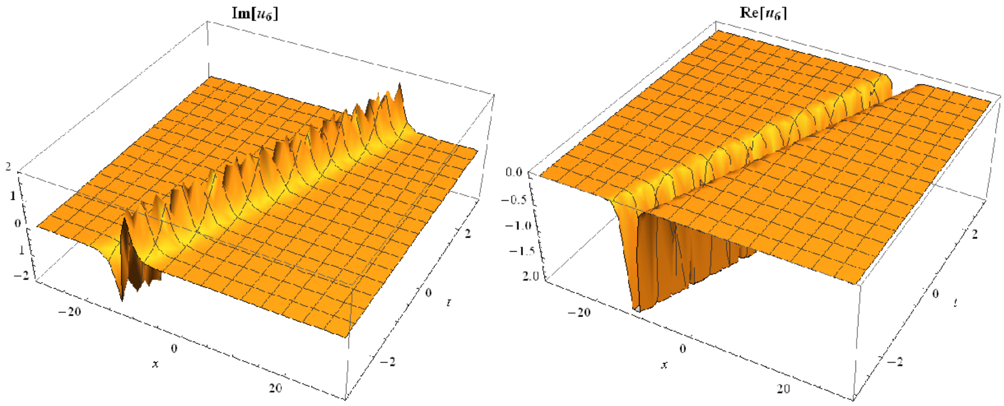
$\alpha = 4$, $k = 0.5$, $c = 1.5$



(c)

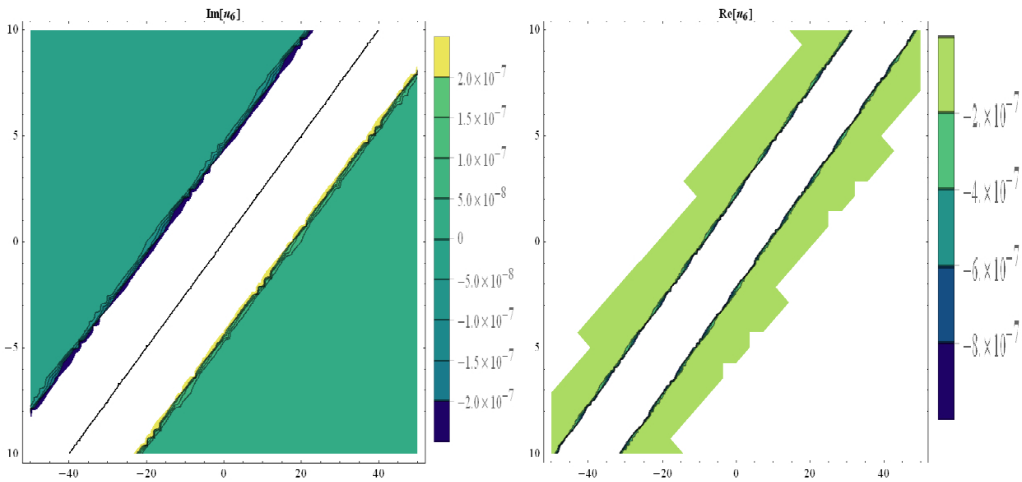
Fig.10: (c) represent 2D plots of Eq. (22) for $\alpha = 4$,

$k = 0.5$, $c = 1.5$, $t = 0.1$



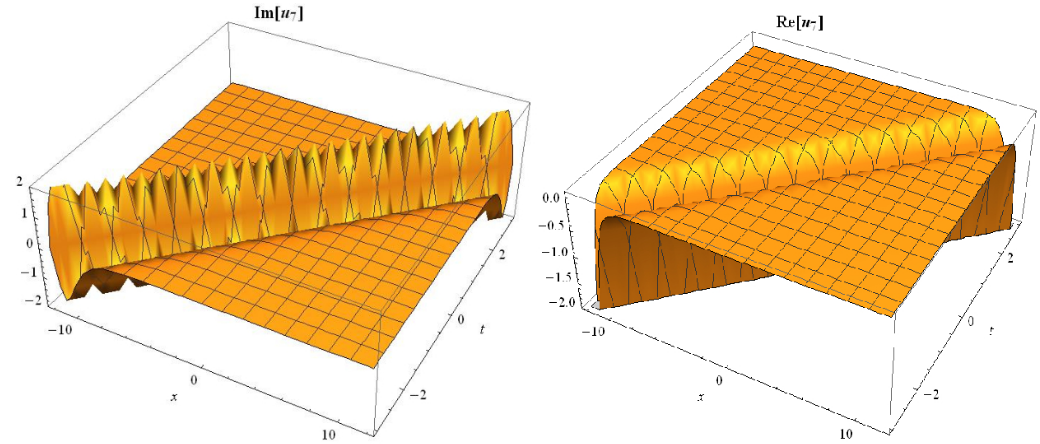
(a) (b)

Fig.11: (a) and (b) represent 3D plots of Eq. (23) for $\alpha = 6$, $k = -1$, $c = -4$.



(c) (d)

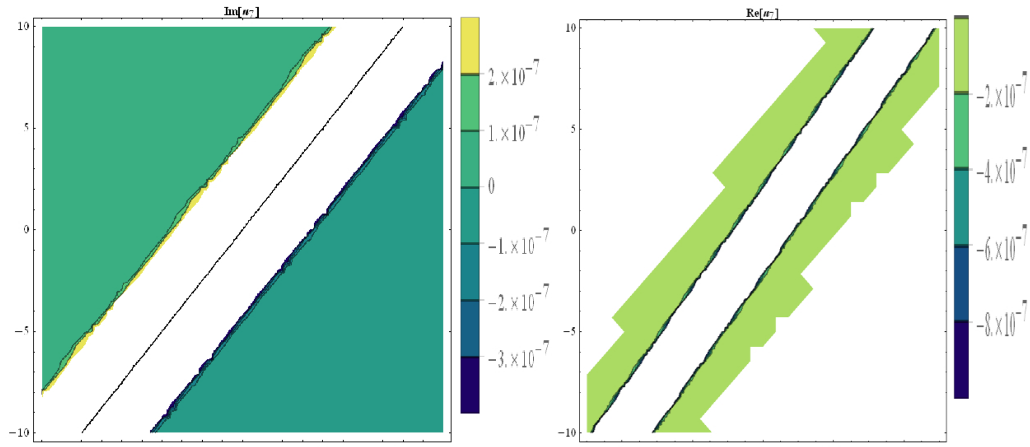
Fig.12: (c) and (d) represent contour plots of Eq.(23) for $\alpha = 4$, $k = 1$, $c = 3$.



(a) (b)

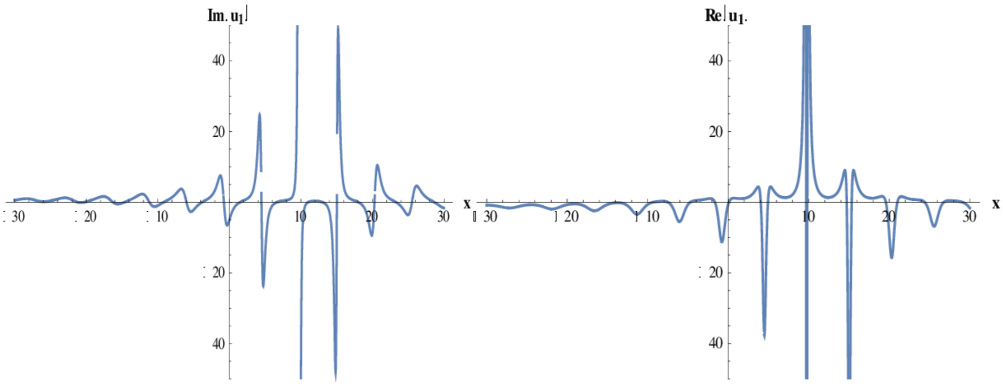
Fig.13: (a) and (b) represent contour plots of Eq. (24) for **α** = 6.

k = -1, c = -4.



(c) (d)

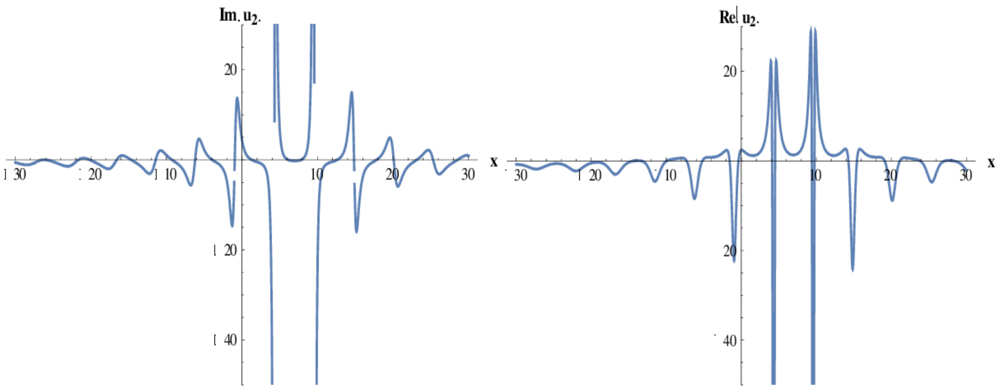
Fig.14: (c) and (d) represent contour plots of Eq. (24) for **α** = 6 k = -1, c = -4.



(a) (b)

Fig.15: (a) and (b) represent 2D plots of Eq. (18) for

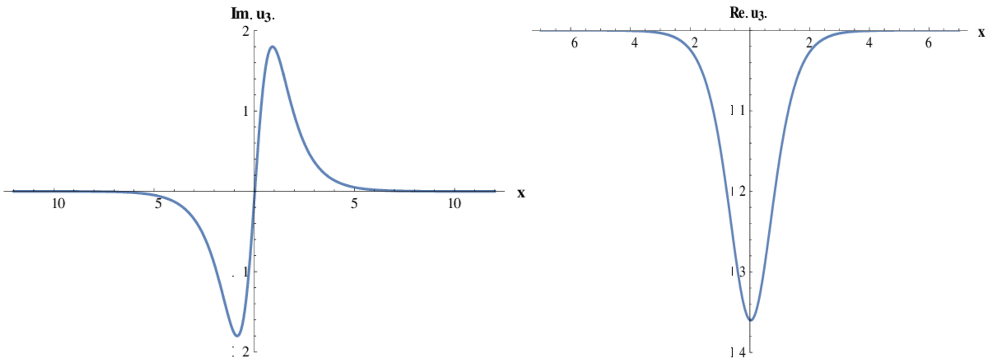
α = 4, t =0.3.



(c) (d)

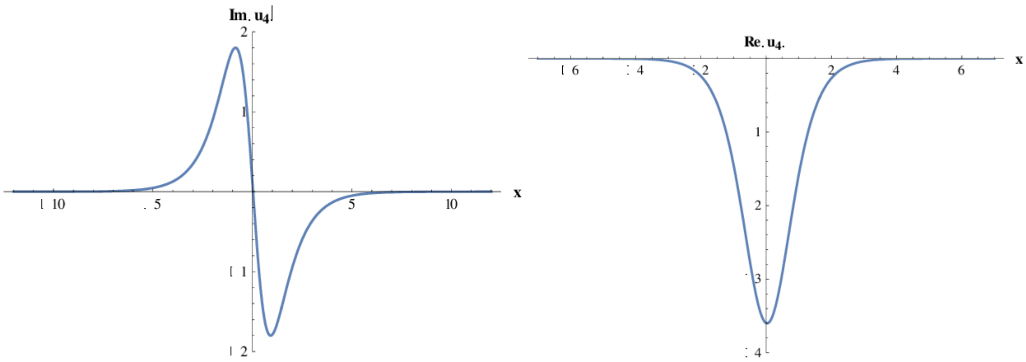
Fig.16 :(c) and (d) represent 2D plots of Eq. (19) for

α = 4, t = 0.2.



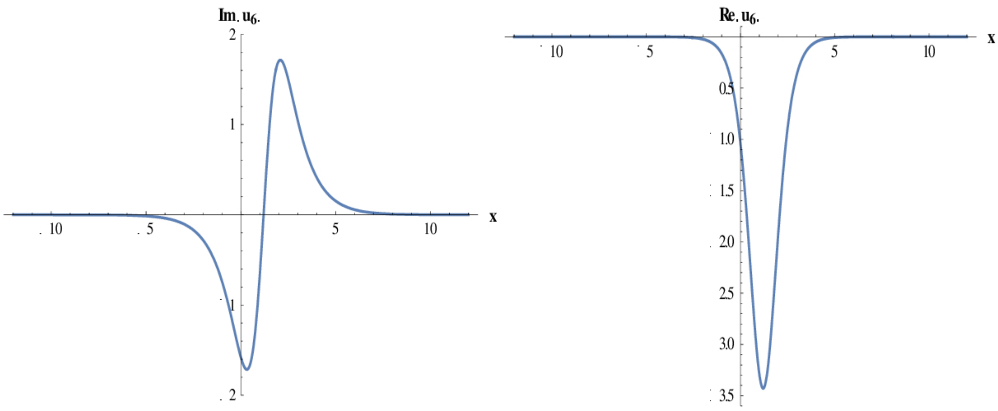
(a) (b)

Fig.17: (a) and (b) represent 2D plots of Eq. (20) for $\alpha = 4$, $k = 1$, $t = 0.01$.



(c) (d)

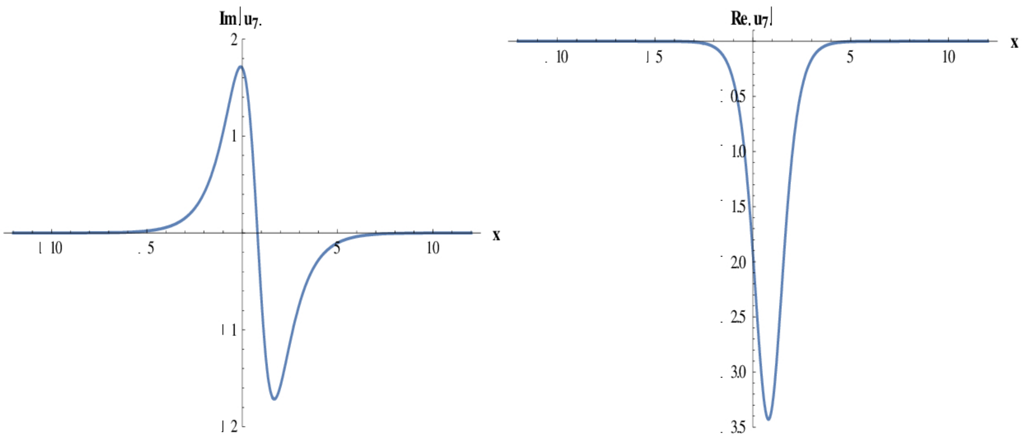
Fig.18: (c) and (d) represent 2D plots of Eq. (21) for $\alpha = 4, k = 1$, $c = 3$, $t = 0.01$.



(a) (b)

Fig.19: (a) and (b) represent 2D plots of Eq. (23) for

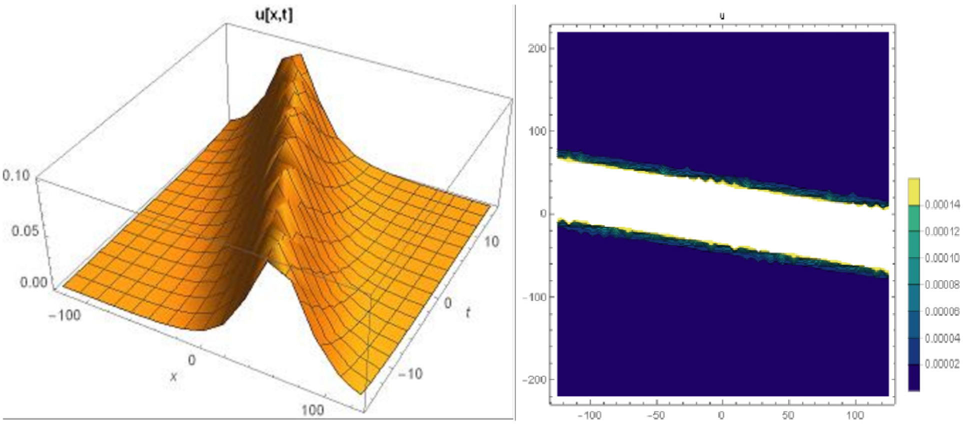
$\alpha = 6,\ c = 4,\ k = 1,\ t = 0.3.$



(c) (d)

Fig.20: (c) and (d) represent 2D plots of Eq. (24) for

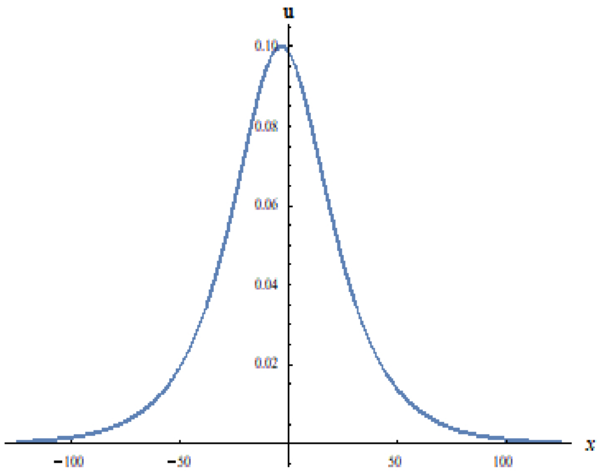
$\alpha = 6,\ k = -1,\ c = 4,\ t = 0.2.$



(a) (b)

Fig.21: (a) and (b) represent 3D and contour plots of Eq. (28) for

B_1 = 0.1, c = 0.2



(c)

Fig.22: (c) represent 2D plots of Eq. (28) for

B_1 = 0.1, c = 0.2, t = 0.9

4 Conclusions

In compendium, in this paper we proposed a SGEMs that is much common than the current classical sine-Gordon method. This new approach is used to prepare various traveling wave solution of the modified α-equation and modified Vakhnenko-Parkes models. Several type of solutions, namely the new complex and mixed dark-bright soliton, the new dark soliton, conjugate mixed dark-bright soliton, singular soliton have been provided. Some of these solutions are new and for instance the soliton ones are used for the transmission of data. In all these solutions, α, k, and c are licentious nonzero constants. The expressions of $u_i(x,t)$ with $i = 1, 2,$ ....8, are acquired from the solution $u_i(x,t)$ though Eq.(18-24) and Eq.(28) these solutions are singular solitons and solitary wave soliton of the modified α-equation and modified Vakhnenko-Parkes model. They have identical shape to those in figs.1-22 for exact value of parameters nevertheless, the 2D and 3D-dimensional representations of few of these solution different enthralling aspect of this work is that the current method the SGEMS can be employed to recover the solution in [18]-[24] and [28] with the classical sine-Gordon expansion also to solve other variant of nonlinear equations.

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